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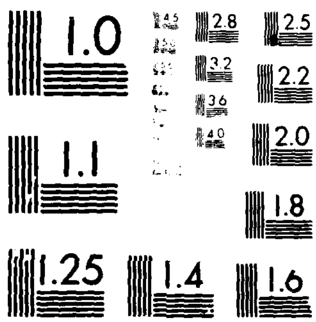


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# DISTRIBUTION-FREE TOLERANCE BOUNDS FOR THE LIFETIME OF AN M-OUT-OF-K SYSTEM OF COMPONENTS.

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10 K. M. LAL/SAXENA


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## Abstract

Distribution-free tolerance bounds are obtained for the life time of an m-out-of-k system of unlike components. It is assumed that the life time distributions of the k components belong to a stochastically ordered family of distributions. Two criteria are used to determine the tolerance bounds. These criteria are extensions of the criteria used in a single population literature.

## 1. Introduction

Reliability studies of series, parallel or complex-systems based on component test data is of considerable interest and practical significance. Sometimes it may be much more economical to test components of a system seperately rather than testing the whole system or it may be that the reliability-testing of the whole system cannot be done without destroying the system. For such situations, working with component test data, a number of authors have considered the problem of lower confidence bounds for a series or parallel system (see Mann (1974), Mann and Grubbs (1974) and the references therein.) The problem discussed in this paper is different in three respects from the confidence bound problem. Instead of confidence bounds for the reliability of a system for a specified time, we consider tolerance bounds for the life time distribution of the system. Usually lower tolerance bounds are of much interest. These tolerance bounds can then be used to obtain confidence bounds for quantiles of the life time distribution of the system (see Conover (1971) p. 118). Since a series system and a parallel system are special cases of an m-out-of-k system (see Barlow and Proschan (1975)), we consider the tolerance bounds for an m-out-of-k system. Such a system is of interest by itself also, as a redundant system. The system then has k components but needs only m to function properly. Lastly the assumption of exponential life time distributions for the components is replaced by a much weaker assumption that the life time distributions of the components belong to a stochastically ordered family of sitributions, that is there is no crossover between any two distribution functions. This assumption of "no crossover" is satisfied by many families of distributions. In particular, the assumption holds if the lifetime distributions for the components are all exponential with different scale parameters or if these life time distributions are Weibull with same shape parameter and different scale parameters or same scale parameter and different shape parameters.

Let $T_1,\dots,T_k$ denote the lifetimes of the k components of an m-out-of-k system with continuous distribution functions $F_1,\dots,F_k$ respectively. Then the lifetime of the system is $T_{(k-m+1)}$, the (k-m+1)st smallest of the $T_i$'s. Let $H_{m,k}$ denote the distribution function of $T_{(k-m+1)}$. We assume that the $F_i$'s do not have any crossovers, i.e., there is a permutation of $(1,2,\dots,k)$, let it be denoted by $((1),\dots,(k))$, such that $F_{(1)}(x) \le \cdots \le F_{(k)}(x)$ for all x. A procedure for the distribution-free tolerance bounds for $H_{m,k}$ is described below.

<u>Procedure</u>. Let $T_{i,r,n}$ denote the waiting time for the rth failure among n prototypes of the ith component, put on test simultaneously, all with distribution function $F_i$ $i=1,\dots,k$. Let the waiting times from the k seperate experiments be ordered as

$$T_{(1);r,n} \le \cdots \le T_{(k);r,n}.$$

For $r \le s$, $i \le j$ (with at least one strict inequality) we define a tolerance interval

$$I_{i,j} = (T_{(i);r,n}, T_{(j);s,n}). \tag{1.1}$$

Adopting the convention that $T_{(0);r,n} = 0$ and $T_{(k+1);s,n} = +\infty$ for any r and s, we can write a lower tolerance interval and an upper tolerance interval as

$$I_{i,k+1} = (T_{(i);r,n}, \infty), \tag{1.2}$$

$$I_{0,j} = (0, T_{(j);s,n}). \tag{1.3}$$

The motivation for considering the above procedure is the following. Suppose for some $\lambda$, $0<\lambda<1$, the $\lambda$-quantiles for the life times of the k components were known as $q_1,\ldots,q_k$, and we decide to use $q_{(k-m+1)}$, the (k-m+1)st smallest of these quantiles, for the lower tolerance bound for $H_{m,k}$. Then the probability that the life time of an m-out-of-k system will exceed $q_{(k-m+1)}$ is $1 - H_{m,k}(q_{(k-m+1)})$. It can be shown that an upper bound for $H_{m,k}(q_{(k-m+1)})$ is $G(F_{(m)}(q_{(k-m+1)});\ 1,\ m) = G(\lambda;1,m) = 1-(1-\lambda)^m$ (G is defined later). Thus if $\lambda \leq 1 - \beta^{1/m}$ where $\beta$ is preassigned, the probability that the life time of an m-out-of-k system will exceed $q_{(k-m+1)}$ is at least $\beta$. Since in practice we do not know $q_i$'s, they are estimated by $T_{i;r,n}$ and then $T_{(k-m+1);r,n}$ estimates $q_{(k-m+1)}$.

We shall use two criteria for the determination of n,r,s,i and j for given m and k. These criteria are extensions of the criteria used in the single population literature (see Guttman (1970)) and were used by Saxena (1976). Let $P(I_{i,j} \mid H_{m,k})$ denote the probability coverage of the tolerance interval $I_{i,j}$ by $H_{m,k}$. Then $P(I_{i,j} \mid H_{m,k})$ is a random variable whose distribution depends on $\underset{\sim}{F}$, where $\underset{\sim}{F} = (F_1,\ldots,F_k)$. Let $\Omega$ be the set of all k-tuples $\underset{\sim}{F}$ in which no two $F_i$'s have any cross overs.

Criterion A. An interval $I_{i,j}$ is a $\beta$-expectation tolerance interval for $H_{m,k}$ if

$$\inf_{\Omega} E_{\underset{\sim}{F}}(P(I_{i,j} \mid H_{m,k})) \geq \beta \ . \tag{1.4}$$

Criterion B. An interval $I_{i,j}$ is a $\beta$-content tolerance interval for $H_{m,k}$ with confidence level $\gamma$ if

$$\inf_{\Omega} P_{\underset{\sim}{F}}(P(I_{i,j} \mid H_{m,k}) \geq \beta) \geq \gamma . \tag{1.5}$$

Note that from (1.2) and (1.5) we can obtain also a lower confidence bound for the $(1-\beta)$ quantile of $H_{m,k}$. It is $T_{(i);r,n}$ with confidence coefficient at least $\gamma$, if i,r,n are chosen to satisfy (1.5).

In the sequel we use the following notation: Let $g(x;r,n)$ given by

$$g(x;r,n) = \frac{\Gamma(n+1)}{\Gamma(r)\Gamma(n-r+1)} x^{r-1}(1-x)^{n-r+1-1}, \ 0 < x < 1,$$

denote the density function of a beta distribution with parameters r and n-r+1 and let the corresponding distribution function be $G(x;r,n)$. Then $G(G(x;r,n);i,j)$ is the cdf of the ith order statistic in a sample of size j from a beta distribution with parameters r and n-r+1.

Lower bounds for the infima in (1.4) and (1.5) are obtained in Theorems 1 and 2 and their corollaries in the Appendix. Using those bounds the choices for i,j,r,s and n for the intervals (1.1)-(1.3) are discussed below in Section 2.

## 2. Choices for i, j, r and s.

Whether we want a tolerance interval of a given expected coverage (Criterion A) or a tolerance interval whose probability coverage exceeds a preassigned $\beta$ (Criterion B), clearly it is desirable to have the intervals as "short" as possible. This is done by taking i and r as large as possible and j and s as small as possible. In view of the restrictions under which the bounds have been obtained in Theorems 1 and 2 in the Appendix we take $i=j=k-m+1$. Then the intervals are

$$I_{k-m+1,k-m+1} = (T_{(k-m+1);r,n}, T_{(k-m+1);s,n}), \tag{2.1}$$

$$I_{k-m+1,k+1} = (T_{(k-m+1);r,n}, \infty), \tag{2.2}$$

$$I_{o,k-m+1} = (0,T_{(k-m+1)}) . \tag{2.3}$$

For example, to get a lower tolerance bound on a 2-out-of-4 system we first find the rth failure times among n prototypes of each of the four components and then take the third smallest of these four failure times. In the rest of this section we consider the choices for r and s for the two criteria seperately.

Criterion A. Let $Z_{(i),j}(m,n)$ denote the ith order statistic in a sample of size j from a population with distribution function $G(x;m,n)$ . Then it is easy to see that the lower bounds for expected coverages of intervals (2.1)-(2.3), from results in the Appendix, can be expressed in terms of the moments of $Z_{(1),j}(m,n)$ in the following manner:

$$\inf_{\Omega} E_{\underset{\sim}{F}} P(I_{k-m+1,k-m+1} \mid H_{m,k}) \geq EZ^{k-m+1}_{(1),m-1}(s,n) + EZ^{m}_{(1),k-m+1}(n-r+1,n) \tag{2.4}$$

$$\inf_{\Omega} E_{\underset{\sim}{F}} P(I_{k-m+1,k+1}) \geq EZ^{m}_{(1),k-m+1}(n-r+1,n), \tag{2.5}$$

$$\inf_{\Omega} E_{\underset{\sim}{F}} P(I_{0,k-m+1}) \geq EZ^{k-m+1}_{(1),m}(s,n) \ . \tag{2.6}$$

Next we have to decide the values for r and s so that the right hand side quantities in the above inequalities are larger than $\beta$. For this we observe that $G(x;r,n) \leq G(x;r',n)$ if $r \geq r'$. Hence from Lemma 2.2 of Saxena (1976) it follows that any moment of $Z_{(1),j}(m,n)$, for fixed j and n, increases with m. So the lower bounds in (2.5) and (2.6) decrease when r is increased and s is decreased. Consequently for the one sided interval, the optimum values for r and s are respectively the largest and smallest values which bring the lower bounds (2.5) and (2.6) as close as possible to the preassigned $\beta$ value. For the two sided tolerance interval it is an extremely difficult problem to choose the largest r and the smallest s such that the lower bound (2.4) stays above $\beta$ and at the same time the interval is as "short" (in the sense of expected length) as possible. To overcome this difficulty we restrict the choices of r and s to those values which make the expected coverage of the two tails outside the interval as close to $(1-\beta)/2$ as possible. This requires that we choose the smalelst s and the largest r so that

$$EZ^{k-m+1}_{(1),m}(s,n) \geq (1+\beta)/2 \text{ and } EZ^{k-m+1}_{(1),m}(n-r+1,n) \geq (1+\beta)/2. \tag{2.7}$$

The moments of $Z_{(1),j}(m,n)$ needed for solving for r and s have to be evaluated by numerical integration. But this numerical integration can be avoided, since for all practical purposes, a lower bound on these moments, given in the lemma below, works as well as the exact value. A few exact values and lower bounds tabulated in Table 1, indicate a difference of less than one per cent.

Lemma 2.1. For any i and j

$$\frac{n!(i+r-1)!}{(r-1)!(n+i)!} \geq EZ^{i}_{(1),j}(r,n) \geq \frac{n!(i+r-1)!}{(r-1)!(n+i)!}$$

$$- \frac{j-1}{\sqrt{2j-1}} \left( \frac{n!(2i+r-1)!}{(r-1)!(n+2i)!} - \left( \frac{n!(i+r-1)!}{(r-1)!(n+i)!} \right)^{2} \right)^{\frac{1}{2}} . \tag{2.8}$$

Proof. Let $g_j(z)$ denote the pdf of $Z_{(1),j}(r,n)$. Then for $j > j', g_j(z)/g_{j'}(z)$ is a decreasing function of z and therefore

$$EZ^{i}_{(1),j}(r,n) \leq EZ^{i}_{(1),1}(r,n) = \frac{n!(i+r-1)!}{(r-1)!(n+i)!} .$$

For the lower bound the proof is based on the approach of Sugiura (1962). The steps of the proof are similar to David (1970), p.54, and are omitted.

Large n: If $r/n \to \lambda$ as $n \to \infty$, it is seen that both the upper and the lower bounds for $EZ^{i}_{(1),j}(r,n)$ in (2.8) coverge to $\lambda^{i}$. So for large n, even an approximation for the lower bound suffices as an approximation for the exact value, i.e.,

$$EZ^{i}_{(1),j}(rn,) \approx \lambda^{i} + i\lambda^{i-1}[-(j-1)(\lambda(1-\lambda)/n(2j-1))^{\frac{1}{2}}+(i-1-i\lambda-\lambda)/2n] \tag{2.9}$$

Criterion B. From results in the Appendix, the lower bounds for the confidence levels for the $\beta$-content tolerance intervals can be written as

$$\inf_{\Omega} P_{\underset{\sim}{F}}(P(I_{k-m+1,k-m+1} \mid H_{m,k}) \geq \beta)$$
$$\geq [G(1-((1+\beta)/2)^{1/m};r,n)]^{k-m+1} + [1-G(((1+\beta)/2)^{1/(k-m+1)};s,n)]^{m}-1, \tag{2.10}$$

$$\inf_{\Omega} P_{\underset{\sim}{F}}(P(I_{0,k-m+1} \mid H_{m,k}) \geq \beta) \geq [1-G(\beta^{1/(k-m+1)};s,n)]^{m}, \tag{2.11}$$

$$\inf_{\Omega} P_{\underset{\sim}{F}}(P(I_{k-m+1,k+1} \mid H_{m,k}) \geq \beta) \geq [G(1-\beta^{1/m};r,n)]^{k-m+1} . \tag{2.12}$$

Since $G(x;r,n)$ is a nonincreasing function of $r$, the lower bounds in (2.11) and (2.12) are decreased when $s$ is decreased and $r$ is increased. Thus the optimum values for $r$ and $s$ are the largest and the smallest values respectively which bring the bounds (2.11) and (2.12) as close to $\gamma$ as possible. For the two sided interval we consider those values of $r$ and $s$ optimum which give a probability of, at most $(1-\gamma)/2$, for the probability content of each tail outside the interval to be at most $(1-\beta)/2$. This is done by choosing the smallest $s$ and the largest $r$ so that

$$G(((1+\beta)/2)^{1/(k-m+1)};s,n) \leq 1-(1+\gamma)/2)^{1/m}, \tag{2.13}$$

and

$$G(1-((1+\beta)/2)^{1/m};r,n) \geq ((1+\gamma)/2)^{1/(k-m+1)} . \tag{2.14}$$

Large n: When n is sufficiently large the normal approximation $\Phi((-r+1+nx)/(nx(1-x))^{\frac{1}{2}})$ for $G(x;r,n)$ can be used for all three intervals.

## 3. Numerical Examples

We consider numerical examples for criteria A and B for the lower tolerance intervals.

Criterion A: Consider a 2-out-of-4 system ($m=2,k=4$). Let $\beta=.8$, $n=40$. For a lower tolerance interval choose the largest r so that $EZ^2_{(1),3}(41-r,40) \simeq .8$. From the table we find that $EZ^2_{(1),3}(38,40)$ $=.7971$. Thus a .8-expectation lower tolerance interval is $(T_{(3);3,40},\infty)$. The approximation (2.9) gives the value .8000 for $EZ^2_{(1),3}(38,40)$.

Criterion B. Consider a 2-out-of-4 system ($m=2,k=4$). Let $\beta=.8$, $\gamma=.8$, $n=40$. Then using (2.12) and the incomplete beta function tables by Pearson (1968) we see that the largest value of r for which

$$G(1-\beta^{1/m};r,n) \geq \gamma$$

is $r=2$. Thus the .8-content lower tolerance interval with confidence level .8 is $(T_{(3);2,40},\infty)$.

## 4. Concluding Remarks

The goal of this paper is to obtain tolerance bounds for the lifetime distribution of an m-out-of-k system based on component test data. The procedure discussed is not recommended when it is practically feasible to test a system as a whole. But situations where it is not possible to test a whole system are numerous and statistical procedures based on component test data have been extensively discussed by various authors cited in section 1. In most of the previous works the emphasis is on exponential distributions and confidence bounds for reliability to an assigned time. Tolerance bounds have not been discussed in previous works. The only assumption made about the lifetime distributions of the components is that they are stochastically ordered. This enables us to work with order statistics and obtain distribution-free bounds.

It should be noted that while determining r and s for the bounds, the extreme values for r and s are 1 and n respectively. If even with such extreme choices for r and s it is not possible to meet the specifications, then a larger sample size has to be used.

## 5. Acknowledgements

The author is thankful to one of the referees amd the Associate Editor for some helpful comments.

## 6. Appendix

<u>Theorem 6.1</u> For the tolerance $I_{i,j}$ given by (1.1), if $i \leq k-m+1 \leq j$, then

$$\inf_{\Omega} E_{\underset{\sim}{F}} P(I_{ij}) H_{m,k} \geq \int_0^1 G(y;k-m+1,j)\,d[G(G(y;s,n);1,k-j+1)]$$

$$- \int_0^1 G(y;k-i-m+2,k-i+1)\,d[G(G(y;r,n);i,i)]. \tag{6.1}$$

Proof. From (1.1) we have

$$E_{\underset{\sim}{F}} P(I_{ij} \mid H_{m,k}) = E_{\underset{\sim}{F}} H_{m,k}(T_{(j);s,n}) - E_{\underset{\sim}{F}} H_{m,k}(T_{(i);r,n})$$

$$= E_1 - E_2 \quad \text{say} . \tag{6.2}$$

A lower bound on the infimum of (6.2) over $\Omega$ is obtained by replacing $E_1$ by a lower bound and $E_2$ by an upper bound. Corresponding to a vector $\underset{\sim}{F} \in \Omega$ define

$$\underset{\sim}{F}^1(i) = (\underbrace{F_{(i)},\ldots,F_{(i)}}_{i},1,\ldots,1) \tag{6.3}$$

and

$$\underset{\sim}{F}_0(i) = (\underbrace{0,\ldots,0}_{i-1},\ F_{(i)},\ldots,F_{(i)}) . \tag{6.4}$$

Then using Lemma 2.1 of Saxena (1976) we have

$$H_{m,k}(t) = P_{\underset{\sim}{F}}(T_{(k-m+1)} \leq t) \geq P_{\underset{\sim}{F}_0(k-j+1)}(T_{(k-m+1)} \leq t)$$

$$= G(F_{(k-j+1)}(t);k-m+1,j) .$$

Then

$$E_1 = E_{\underset{\sim}{F}} H_{m,k}(T_{(j);s,n}) \geq E_{\underset{\sim}{F}} G(F_{(k-j+1)}(T_{(j);s,n});k-m+1,j)$$

$$\geq E_{\underset{\sim}{F}^1(k-j+1)} G(F_{(k-j+1)}(T_{(j);s,n});k-m+1,j) .$$

Under $\underset{\sim}{F}^1(k-j+1)$, the cdf of $T_{(j);s,n}$ is $G(G(F_{(k-j+1)}(\cdot);s,n);1,k-j+1)$. Therefore

$$E_1 \geq \int_0^1 G(y;k-m+1,j)d[G(G(y;s,n);1,k-j+1)]. \tag{6.5}$$

Working similarly for an upper bound for $E_2$ we have

$$E_2 \leq \int_0^1 G(y;k-i-m+2,k-i+1)d[G(G(y;r,n);i,i)]. \tag{6.6}$$

Now the theorem is proved using bounds (6.5) and (6.6) in (6.2).

Corollary 6.1 For the one sided tolerance intervals $I_{i,k+1}$ and $I_{0,j}$ given by (1.2) and (1.3), if $i \leq k-m+1 \leq j$, then

$$\inf_{\Omega} E_{\underset{\sim}{F}} P(I_{i,k+1} \mid H_{m,k}) \geq 1- \int_0^1 G(y;k-i-m+2,k-i+1)d[G(G(y;r,n);i,i)], \tag{6.7}$$

$$\inf_{\Omega} E_{\underset{\sim}{F}} P(I_{0,j} \mid H_{m,k}) \geq \int_0^1 G(y;k-m+1,j)d[G(G(y;s,n);1,k-j+1)]. \tag{6.8}$$

The proofs are omitted.

Theorem 6.2 For the tolerance interval $I_{i,j}$ given by (1.1), if $i \leq k-m+1 \leq j$, then

$$\begin{aligned}\inf_{\Omega} P_{\underset{\sim}{F}}(P(I_{i,j} \mid H_{,mk}) \geq \beta) \\ &\geq G(G(G^{-1}((1-\beta)/2;k-i-m+2,k-i+1);r,n);i,i) \\ &- G(G(G^{-1}((1+\beta)/2;k-m+1,j);s,n);1,k-j+1).\end{aligned} \tag{6.9}$$

Proof. We can write

$$\begin{aligned}P_{\underset{\sim}{F}}(P(I_{i,j} \mid H_{m,k}) \geq \beta) &= P_{\underset{\sim}{F}}(H_{m,k}(T_{(j);s,n}) - H_m(T_{(i);r,n}) \geq \beta) \\ &\geq P_{\underset{\sim}{F}}(H_{m,k}(T_{(i)r,n}) \leq (1-\beta)/2) \\ &- P_{\underset{\sim}{F}}(H_{m,k}(T_{(j);s,n}) \leq (1+\beta)/2) \\ &= P_1((1-\beta)/2) - P_2((1+\beta)/2, \text{ say.}\end{aligned} \tag{6.10}$$

Now we get a lower bound for the infimum over $\Omega$ by replacing $P_1(\cdot)$ by a lower bound and $P_2(\cdot)$ by an upper bound. From lemma 2.1 of Saxena (1976) we have

$$H_{m,k}(t) \leq P_{\underset{\sim}{F}^1(k-i+1)}(T_{(k-m+1)}\leq t)$$

$$= G(F_{(k-i+1)}(t);k-i-m+2,k-i+1).$$

Then

$$P_1((1-\beta)/2) \geq P_{\underset{\sim}{F}}(G(F_{(k-i+1)}(T_{(i);r,n});k-i-m+2,k-i+1)\leq(1-\beta)/2)$$

$$= P_{\underset{\sim}{F}}(F_{(k-i+1)}(T_{(i);r,n})\leq a), \qquad (6.11)$$

where $a = G^{-1}((1-\beta)/2;k-i-m+2,k-i+1)$. From theorem 2.2 (b) of Saxena (1976)

$$\inf_{\Omega} P_{\underset{\sim}{F}}(F_{(k-i+1)}(T_{(i);r,n})\leq a) = G(G(a;r,n);i,i).$$

Hence from (6.11)

$$\inf_{\Omega} P_1((1-\beta)/2)\geq G(G(a;r,n);i,i). \qquad (6.12)$$

Similarly we can show that

$$\sup_{\Omega} P_2((1+\beta))/2)\leq G(G(b;s,n);1,k-j+1), \qquad (6.13)$$

where $b = G^{-1}((1+\beta)/2;k-m+1,k-j+1)$. Now using (6.12) and (6.13) in (6.9), the theorem is proved.

Corollary 6.2. For the one sided tolerance intervals $I_{i,k+1}$ and $I_{0,j}$ given by (1.2) and (1.3), if $i \leq k-m+1 \leq j$, then

$$\inf_{\Omega} P_{\underset{\sim}{F}}(P(I_{i,k+1} \mid H_{m,k})\geq\beta)\geq G(G(G^{-1}((1-\beta);k-i-m+2,k-i+1);r,n);i,i), \qquad (6.14)$$

$$\inf_{\Omega} P_{\underset{\sim}{F}}(P(I_{0,j} \mid H_{m,k})\geq\beta)\geq 1-G(G(G^{-1}(\beta;k-m+1,j);s,n);1,k-j+1). \qquad (6.15)$$

The proofs are omitted.

Table 1

Expected values of $Z^{i}_{(1),j}(r,n)$ and the lower bounds: First entry is the exact value of $EZ^{i}_{(1),j}(r,n)$ and the second entry is the lower bound given by (2.8); ** in the second entry indicate that the lower bound (2.8) is equal to the exact value.

| n: | 10 | | 20 | | | | 30 | | | | 40 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| r: | 10 | 9 | 20 | 19 | 18 | 17 | 30 | 29 | 28 | 27 | 40 | 39 | 38 | 37 | 36 |
| (i,j) | | | | | | | | | | | | | | | |
| (1,1) | .9091 | .8182 | .9524 | .9048 | .8571 | .8095 | .9678 | .9355 | .9032 | .8710 | .9756 | .9512 | .9268 | .9024 | .8780 |
| | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** |
| (1,2) | .8658 | .7567 | .9291 | .8708 | .8158 | .7627 | .9519 | .9121 | .8745 | .8380 | .9636 | .9334 | .9048 | .8771 | .8499 |
| | .8611 | .7539 | .9262 | .8686 | .8141 | .7612 | .9417 | .9104 | .8731 | .8386 | .9619 | .9320 | .9036 | .8760 | .8489 |
| (2,1) | .8333 | .6818 | .9091 | .8225 | .7403 | .6623 | .9375 | .8770 | .8185 | .7621 | .9524 | .9059 | .8606 | .8165 | .7735 |
| | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** |
| (1,3) | .8379 | .7207 | .9139 | .8504 | .7921 | .7366 | .9404 | .8979 | .8578 | .8194 | .9556 | .9225 | .8919 | .8626 | .8342 |
| | .8348 | .7186 | .9118 | .8488 | .7904 | .7346 | .9398 | .8966 | .8565 | .8180 | .9543 | .9214 | .8909 | .8615 | .8329 |
| (2,2) | .7576 | .5844 | .8658 | .7623 | .6708 | .5878 | .9073 | .8338 | .7673 | .7055 | .9292 | .8723 | .8203 | .7713 | .7247 |
| | .7520 | .5824 | .8612 | .7596 | .6689 | .5865 | .9036 | .8314 | .7655 | .7040 | .9262 | .8702 | .8185 | .7697 | .7233 |
| (3,1) | .7692 | .5769 | .8696 | .7510 | .6437 | .5471 | .9091 | .8239 | .7441 | .6697 | .9032 | .8638 | .8006 | .7405 | .6836 |
| | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** |
| (1,4) | .8174 | .6958 | .9026 | .8359 | .7757 | .7189 | .9336 | .8878 | .8461 | .8066 | .9496 | .9147 | .8829 | .8527 | .8234 |
| | .8150 | .6919 | .9009 | .8338 | .7725 | .7146 | .9323 | .8862 | .8440 | .8038 | .9486 | .9135 | .8813 | .8505 | .8208 |
| (2,3) | .7102 | .5306 | .8378 | .7271 | .6323 | .5482 | .8875 | .8082 | .7383 | .6744 | .9139 | .8522 | .7971 | .7460 | .6980 |
| | .7073 | .5278 | .8349 | .7250 | .6298 | .5448 | .8850 | .8063 | .7363 | .6720 | .9118 | .8506 | .7954 | .7441 | .6957 |
| (3,2) | .6689 | .4593 | .8089 | .6706 | .5555 | .4575 | .8658 | .7641 | .6756 | .5965 | .8966 | .8164 | .7450 | .6798 | .6197 |
| | .6639 | .4583 | .8035 | .6680 | .5541 | .4564 | .8612 | .7613 | .6737 | .5921 | .8926 | .8138 | .7431 | .6782 | .6183 |
| (4,1) | .7143 | .4945 | .8333 | .6884 | .5632 | .4560 | .8824 | .7754 | .6785 | .5909 | .9091 | .8245 | .7460 | .6732 | .6059 |
| | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** |
| (1,5) | .8014 | .6768 | .8937 | .8248 | .7616 | .7055 | .9274 | .8800 | .8372 | .7969 | .9449 | .9071 | .8760 | .8451 | .8153 |
| | .7984 | .6697 | .8918 | .8213 | .7577 | .6979 | .9261 | .8776 | .8335 | .7920 | .9439 | .9069 | .8733 | .8414 | .8107 |
| (2,4) | .6764 | .4946 | .8174 | .7027 | .6064 | .5220 | .8730 | .7901 | .7104 | .6534 | .9026 | .8379 | .7810 | .7288 | .6800 |
| | .6736 | .4866 | .8150 | .6989 | .6002 | .5133 | .8709 | .7874 | .7143 | .6479 | .9009 | .8358 | .7779 | .7247 | .6749 |
| (3,3) | .6081 | .3978 | .7704 | .6248 | .5084 | .4118 | .8379 | .7291 | .6375 | .5574 | .8747 | .7883 | .7136 | .6466 | .5857 |
| | .6061 | .3932 | .7672 | .6225 | .5048 | .4066 | .8349 | .7270 | .6350 | .5541 | .8720 | .7864 | .7115 | .6440 | .5825 |
| (4,2) | .5952 | .3665 | .7576 | .5926 | .4631 | .3592 | .8272 | .7016 | .5966 | .5064 | .8658 | .7650 | .6778 | .6006 | .5315 |
| | .5913 | .3659 | .7520 | .5906 | .4620 | .3581 | .8220 | .6989 | .5949 | .5052 | .8611 | .7622 | .6760 | .5992 | .5302 |
| (5,1) | .6667 | .4286 | .8000 | .6333 | .4956 | .5220 | .8571 | .7311 | .6203 | .5234 | .8889 | .7879 | .6962 | .6134 | .5386 |
| | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** | ** |

## References


[1] Barlow, R. E. and Proschan, F. (1975). Statistical Theory of Reliability and Life Testing-Probability Models. New York: Holt, Rinehart and Winston, Inc.

[2] Conover, W. J. (1971). Practical Nonparametric Statistics. New York: John Wiley and Sons, Inc.

[3] David, H. A. (1970). Order Statistics. New York: John Wiley and Sons, Inc.

[4] Guttman, I.(1970. Statistical Tolerance Regions. London: Griffin Press

[5] Mann, Nancy R. (1974). Approximately optimum confidence bounds on series and parallel system reliability for systems with binomial subsystem data. IEEE Transactions on Reliability, 23, 295-303.

[6] Mann, Nancy R. and Grubbs, Frank E. (1974). Approximately optimum confidence bounds for system reliability based on component test data. Technometrics, 16, 335-347.

[7] Pearson, E. S., and Johnson, N. L. (1968). Tables of the Incomplete Beta-Function London: Cambridge University Press.

[8] Pledger, G., and Proschan, F. (1971). Comparisons of order statistics and of spacings from heterogeneous distributions. Optimization Methods in Statistics 89-113. New York: Academic Press, Inc.

[9] Saxena, K. M. Lal (1976). Distribution-free tolerance intervals for stochastically ordered distributions. Ann. Statist., 4,1210-1218.

[10] Sen, P. K. (1970). A note on order statistics for heterogeneous distributions. Ann. Math. Statist.,41,2137-2139.

[11] Sugiura, N. (1962). On the orthogonal inverse expansion with an application to the moments of order statistics. Osaka Math. J.,14,253-263.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Distribution-free tolerance bounds are obtained for the life time of an m-out-of-k system of unlike components. It is assumed that the life time distributions of the k components belong to a stochastically ordered family of distributions. Two criteria are used to determine the tolerance bounds. These criteria are extensions of the criteria used in a single population literature.